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Decentralized Event-Triggered Control of Networked Systems–Part 1: Leader-Follower Consensus Under Switching Topologies

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***Abstract*—A decentralized controller that uses event-triggered scheduling is developed for the leader-follower consensus problem under switching communication topologies. To reduce inter-agent communication, a feedback controller is designed based on state estimates of neighboring agents that are updated by scheduled communication. The state estimates are updated when the network topology switches or a decentralized trigger condition is met. The trigger condition is designed such that the control requires reduced inter-agent communication for feedback while still achieving leader-follower consensus under switching topologies. Since the control strategy produces switched dynamics, analysis is provided to show that Zeno behavior is avoided by developing a positive constant lower bound on the minimum inter-event interval. A Lyapunov-based convergence analysis is also provided to indicate asymptotic convergence of the developed control methodology.**

I. INTRODUCTION

To increase efficiency and speed, a group of robots can cooperate to perform a task, wherein the coordination and control of the agents rely on network communication. Centralized communication architectures facilitate global behaviors by networked agents; however, such strategies exacerbate network congestion when compared to decentralized approaches. In decentralized communication architecture, each agent uses only local communication (e.g., one-hop neighbors) (cf. [1]–[4]). A network leader can be included in decentralized architectures where only a subset of the agents communicate with and follow the leader (cf. [1]–[4]). However, most decentralized network control approaches (cf. [5]–[7]) rely on continuous inter-agent communication for control feedback.

To further reduce bandwidth usage, real-time scheduling methods, also called event-triggered approaches (cf. [8], [9]), can be applied instead of continuous state feedback. In event-triggered control, the control task is executed when a triggering condition is met, which is typically when the ratio of the norm of some error to the state norm exceeds a predefined threshold. The earliest event-triggered strategies applied to control a multi-agent system are in [10] and [11]. However, the potential bandwidth minimizing advantages are compromised because verifying the event triggering condition requires constant communication. These results were later extended to directed and undirected graphs in [12] and [13], but in these works the triggering condition requires a priori knowledge of the Fiedler value and the final consensus value. These requirements were relaxed in [14] and [15] by designing a new trigger function using the sum of relative states from neighbors. In [16], a time-based triggering function (i.e., a time-dependent threshold) is introduced to ensure asymptotic convergence to a neighborhood of the average consensus value. A similar time-varying threshold is applied in [17] for a directed time-varying communication topology under neighbor-synchronous and asynchronous updating protocols. However, the strategies in [10]–[17] solve the leaderless average consensus problem. The more challenging leader-follower consensus control problem is investigated in [18], but the leader state is assumed to be stationary, which limits applicability; additionally, constant neighbor communication is used to detect the trigger condition, which mitigates the benefits of the event-triggered control strategy.

In practice, unpredictable physical constraints (e.g., equipment failure, environmental factors) can cause intermittent communication. Since these additional discrete events introduce discontinuous dynamics into the system, a switched event-triggered controller for the hybrid system is motivated. However, it is unclear how to directly extend the strategies in the aforementioned results that assume a fixed topology.

A decentralized controller using state-estimate feedback is developed in this paper to reduce communication bandwidth. The estimators are updated by scheduled inter-agent communication. The communication events happen when the network topology switches or the decentralized triggering condition is met. The triggering condition is designed based on a stability analysis and ensures that no communication is necessary between two event times. A convergence analysis shows that the controller requires only intermittent communication while still achieving leader-follower consensus with asymptotic convergence. Part 2 of this work in [19] extends this event-triggered approach to the multiple leader problem for a fixed network topology.

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II. Preliminaries and problem Statement

A. Algebraic Graph Theory Preliminaries

A directed graph $\bar{\mathcal{G}}$ consists of a finite node set $\mathcal{V}$, defined as $\mathcal{V} \triangleq \{1, 2, \cdots, N\}$, and an edge set $\mathcal{E}$, where $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$ is a set of paired nodes. An edge, denoted as (i, j), implies that node j can obtain information from node i, but not vice versa. On the contrary, the graph $\mathcal{G}$ is undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$, and vice versa. The neighbor set of agent i is defined as $\mathcal{N}_i \triangleq \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}, j \neq i\}$.

A directed path is a sequence of edges in a graph. An undirected path of the undirected graph is defined analogously. A undirected graph is connected if there exist a undirected path between any two distinct nodes in the graph. An adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ of the directed graph is given by $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. For the undirected graph, $a_{ij} = a_{ji}$. For both the directed and undirected graph, $a_{ii} = 0$ holds, and furthermore, it is assumed that $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$. The Laplacian matrix of the graph $\mathcal{G}$ is defined as $L = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

B. Dynamics

Consider a network system consisting of N followers and a leader, where the interaction topology is time-varying, and only a small subset of the follower agents have direct accesses to the leader's states. The dynamics of the N followers are described by

$$\dot{x}_i = Ax_i + Bu_i, \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the state and control input of agent i, respectively, $B \in \mathbb{R}^{n\times m}$ is a full column rank matrix, and $A \in \mathbb{R}^{n\times n}$ is a state matrix. The leader, indexed by 0, has dynamics given by

$$\dot{x}_0 = Ax_0, \tag{2}$$

where $x_0 \in \mathbb{R}^n$ denotes the leader's state.

Assumption 1. The dynamics of the follower agents are controllable, or the pair (A, B) is stabilizable.

C. Definition and Assumptions

The time-varying interaction topology of the N followers described in (1) can be modeled by a switched undirected graph $\mathcal{G}_\sigma$, where the piecewise constant switching signal $\sigma : [0, \infty) \to \mathcal{P}$ indicates an underlying graph from a finite set $\mathcal{P} \triangleq \{1, 2, \ldots, M\}$ at time t, such that $\{\mathcal{G}_p : p \in \mathcal{P}\}$ includes all graphs in $\left\{ \underset{t\geq 0}{\cup} \mathcal{G} \right\}$.

Similarly, the time-varying interaction topology of the leader-follower system described in (1)-(2) is modeled by a directed switching graph denoted as $\bar{\mathcal{G}}_\sigma$, which consists of the node set $\mathcal{V} \cup \{0\}$ and the edge set that contains all edges in $\mathcal{G}_\sigma$ and the edges connecting node 0 and the followers that have a directed edge from the leader.

Definition 1. A directed graph is connected if each follower has a directed path from the leader.

Assumption 2. $\bar{\mathcal{G}}_p$ is connected for each $p \in \mathcal{P}$.

Assumption 3. The switching signal σ has a finite number of switches in a finite time interval. Specifically, σ switches at t_q and is invariant during a non-vanishing interval $[t_q, t_{q+1})$, $q = 0, 1, \ldots$, with $t_0 = 0$, $0 < \mu < t_{q+1} - t_q < T$, where $\mu, T \in \mathbb{R}$ are positive constants, and μ is a non-vanishing dwell-time. Additionally, the switching sequence of σ is arbitrary.

D. Conventional Approach and Control Objective

A decentralized controller for the system in (1) and (2) can be developed using conventional continuous feedback such as in [5], for example, as

$$u_i = K \sum_{j\in\mathcal{N}_i} (x_j - x_i) + Kd_i (x_0 - x_i), \; i \in \mathcal{V}, \tag{3}$$

where $K \in \mathbb{R}^{m\times n}$ is the control gain matrix designed in the subsequent analysis, and $d_i = 1$ if agent $i \in \mathcal{V}$ is connected to the leader, $d_i = 0$ otherwise. Note that this control implementation requires continuous state feedback from the neighboring agents.

The leader-follower consensus problem is achieved if the network system described in (1) and (2) satisfies

$$\varepsilon_i \to 0 \quad \text{as} \quad t \to \infty, \quad i \in \mathcal{V}, \tag{4}$$

where $\varepsilon_i \triangleq x_i - x_0 \in \mathbb{R}^n$ represents the leader-follower consensus error for agent i.

III. Development of the Event-triggered Decentralized Controller

To eliminate the need for continuous communication while achieving the control objective, an event-triggered based decentralized control approach is developed. Instead of continuous state feedback, the developed decentralized controller is a piecewise continuous input signal, where inter-agent communication is required only at discrete events. These events include topology switches and triggered events when the decentralized trigger condition is met, and the design of this trigger condition is based on insights from the Lyapunov-based state convergence analysis.

A. Controller Design

Motivated by conventional continuous feedback controller as in (3) and based on the subsequent convergence analysis, a decentralized event-triggered controller for agent $i \in \mathcal{V}$ is designed as

$$u_i = K\hat{z}_i, \tag{5}$$

$$\hat{z}_i = \sum_{j\in\mathcal{N}_i} (\hat{x}_j - \hat{x}_i) + d_i(x_0 - \hat{x}_i), \tag{6}$$

where the followers that are connected to the leader can continuously receive x_0 from the leader. In contrast to the controller in (3), the computation of $\hat{z}_i$ for the controller in (5) only requires the estimates of agent i and its neighboring followers' state estimate (i.e., $\hat{x}_{j\in\mathcal{N}_i}$), instead of using their true states $x_{j\in\mathcal{N}_i}$ via continuous communication. The estimate $\hat{x}_j$ in (6) evolves according to the dynamics

$$\hat{x}_j\left(t_E^j\right) = x_j\left(t_E^j\right) \tag{7}$$

$$\dot{\hat{x}}_j = A\hat{x}_j,\, t\in\left[t_E^j,\, t_{E+1}^j\right),\, j\in\{i\}\cup\mathcal{N}_i \tag{8}$$

$$t_E^j = \begin{cases} t_q, & \text{if } j \text{ is a new neighbor} \\ t_k^j, & \text{otherwise} \end{cases}, \tag{9}$$

for $E,\, k = 0,\, 1,\, 2\ldots$, where $\hat{x}_j$ is updated via x_j communicated from neighboring agent j at its discrete times t_E^j and flows along the leader dynamics during $t\in\left[t_E^j,\, t_{E+1}^j\right)$, for $j\in\mathcal{N}_i$. In (9), t_q is the time when $\bar{\mathcal{G}}_\sigma$ switches, and t_k^j is the event-triggered time of the follower agent j described in Section III-C. Although agent $i\in\mathcal{V}$ does not communicate the estimate $\hat{x}_i$, agent i maintains $\hat{x}_i$ for implementation in (6). The estimate $\hat{x}_i$ is updated continuously with the dynamics in (8) and discretely at time instances described in (7). Therefore, u_i is a piecewise continuous signal, where communication is required when any new one-hop neighbor is connected or when state information is transmitted to, or received from, neighboring agents for estimate updates; otherwise, u_i flows continuously during the inter-event intervals.

Remark 1. In (9), since the link between two follower neighbors is undirected, $j\in\mathcal{N}_i$ implies $i\in\mathcal{N}_j$. That is, mutual communication is conducted at t_q if $j\in\mathcal{N}_i$ is a new neighbor.

B. Dynamics of Estimate Error

Since x_i follows different dynamics from the estimate $\hat{x}_i$ computed by its neighbors, an estimate error $e_i\in\mathbb{R}^n$ characterizing this mismatch is defined as

$$e_i \triangleq \hat{x}_i - x_i,\;\, i\in\mathcal{V},\, t\in\left[t_E^i,\, t_{E+1}^i\right), \tag{10}$$

where e_i is reset to 0 at t_E^i due to the estimate updates. Although x_i and $\hat{x}_i$ are both known for agent i: using $\hat{x}_i$ enables agent i to judge how far a neighbor's estimate of x_i is from its actual state. Using (1), (5), and (8), the time-derivative of (10) can be expressed as

$$\dot{e}_i = A\left(\hat{x}_i - x_i\right) - BK\sum_{j\in\mathcal{N}_i}\left(\hat{x}_j - \hat{x}_i\right) - BKd_i\left(x_0 - \hat{x}_i\right),$$

which can be written in stack form as

$$\dot{e} = \left(I_N\otimes A\right)e + \left(H_\sigma\otimes BK\right)\varepsilon + \left(H_\sigma\otimes BK\right)e, \tag{11}$$

where $e\in\mathbb{R}^{nN}$ denotes $e\triangleq\left[e_1^T, e_2^T, \ldots, e_N^T\right]^T$, I is an identity matrix with denoted dimension, $\otimes$ denotes the Kronecker product, $\varepsilon\in\mathbb{R}^{nN}$ defined as $\varepsilon\triangleq\left[\varepsilon_1^T,\, \varepsilon_2^T,\, \ldots,\, \varepsilon_N^T\right]^T$ is a stack form of ε_i introduced in (4), and the matrix $H_\sigma\in\mathbb{R}^{N\times N}$ is defined as $H_\sigma\triangleq L_\sigma + D_\sigma$, where $D_\sigma\in\mathbb{R}^{N\times N}$ is defined as $D_\sigma\triangleq\operatorname{diag}\left(d_1,\, d_2,\, \ldots,\, d_N\right)$, and $L_{\sigma(t)}\triangleq L\left(t\right)$. Based on Assumption 1, there exists a symmetric positive definite matrix $P\in\mathbb{R}^{n\times n}$ that satisfies the following Riccati inequality

$$PA + A^TP - 2\delta_{\min}PBB^TP + \delta_{\min}I_n < 0, \tag{12}$$

so that the control gain in (3) can be designed as

$$K = B^TP, \tag{13}$$

where $\delta_{\min}\in\mathbb{R}_{>0}$ is defined as

$$\delta_{\min}\triangleq\min\left\{\delta_p \mid p\in\mathcal{P}\right\}, \tag{14}$$

denoting the minimum value of a finite set composed of δ_p, where $\delta_p\in\mathbb{R}_{>0}$ denotes the minimum eigenvalue of H_p and is a positive constant based on Assumption 2 and [20].

C. Communication Mechanism at Event Times

This subsection describes how the communication between follower neighbors proceeds during triggered events.

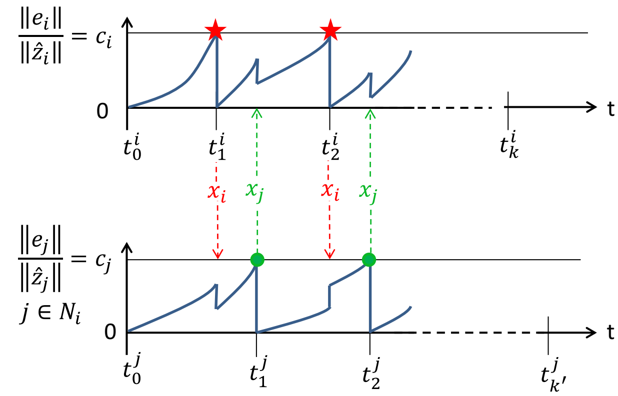

Figure 1. Inter-agent communication mechanism under the developed event-triggered approach. The stars and dots represent the instances when decentralized triggering conditions are satisfied. At the same time, the triggered agents communicate their states over the network to update neighbors' estimates.

In Fig. 1, $\frac{\|e_i\|}{\|\hat{z}_i\|}$ is a decentralized, non-negative, and piecewise continuous signal used to verify the triggering condition. The detailed design of the trigger condition is shown in Section IV. The red stars represent the event-triggered time t_k^i when $\frac{\|e_i\|}{\|\hat{z}_i\|}$ reaches a constant c_i, designed in the subsequent analysis. At t_k^i, x_i is communicated over the network to update the estimate $\hat{x}_i$, used by each neighboring agent $j\in\mathcal{N}_i$. Additionally, $\frac{\|e_i\|}{\|\hat{z}_i\|}$ is reset to zero at t_k^i since the updated estimate has no estimate error. Similarly, at neighbor agent j's event time t_k^j, x_j is communicated over the network to update the estimate $\hat{x}_j$. Since $\|\hat{z}_i\|$ is a decentralized and estimate-based function, verification of the triggering conditions requires no neighbor state information, and hence no communication is required during any inter-event interval (e.g., $\left[t_1^i,\, t_1^j\right)$, $\left[t_1^j,\, t_2^i\right)$, $\left[t_2^i,\, t_2^j\right)$). Additionally, when the network topology switches, newly connected or reconnected neighbors communicate their states simultaneously for the estimate updates.

D. Closed-loop error system

Using (10), a non-implementable form (to facilitate the subsequent analysis) of (5) can be expressed as

$$\begin{aligned} u_i =& K\sum_{j\in\mathcal{N}_i}\left[(x_j-x_i)+(e_j-e_i)\right] \\ &+Kd_i\left(x_0-x_i\right)-Kd_ie_i. \end{aligned} \tag{15}$$

Substituting (15) into the open-loop dynamics in (1) and using the definition in (4) yields the closed-loop error system

$$\begin{aligned} \dot{\varepsilon}_i =& A\varepsilon_i+BK\sum_{j\in\mathcal{N}_i}\left(x_j-x_i\right)+BKd_i\left(x_0-x_i\right) \\ &+BK\sum_{j\in\mathcal{N}_i}\left(e_j-e_i\right)-BKd_ie_i, \end{aligned}$$

where the stack form can be expressed as

$$\dot{\varepsilon}=\left(I_N\otimes A\right)\varepsilon-\left(H_\sigma\otimes BK\right)\varepsilon-\left(H_\sigma\otimes BK\right)e. \tag{16}$$

To facilitate the subsequent analysis, a relation between ε and $\hat{z}$ is developed, where $\hat{z}\triangleq\left[\hat{z}_1,\,\hat{z}_2,\,\ldots,\hat{z}_N\right]^T\in\mathbb{R}^{nN}$ represents the stack form of $\hat{z}_i$ and can be expressed as

$$\hat{z}\triangleq\left(H_\sigma\otimes I_n\right)\left[1_N\otimes x_0-\hat{x}\right], \tag{17}$$

where $\hat{x}\triangleq\left[\hat{x}_1,\,\hat{x}_2,\,\ldots,\hat{x}_N\right]^T\in\mathbb{R}^{nN}$, and 1_N is the ones vector with denoted dimension. Using the relation $\varepsilon_i=\hat{x}_i-e_i-x_0$, the useful expression

$$\hat{x}-1_N\otimes x_0=\varepsilon+e \tag{18}$$

can be obtained. Combining (17) and (18) yields

$$\varepsilon=-\left(H_\sigma^{-1}\otimes I_n\right)\hat{z}-e, \tag{19}$$

where $\hat{z}$ is governed by the dynamics

$$\dot{\hat{z}}=\left(I_N\otimes A\right)\hat{z}, \tag{20}$$

where (2) and (8) were used.

IV. Convergence Analysis

In this section, convergence of leader-follower consensus with the event-triggered controller designed in (5) is examined using a Lyapunov-based analysis. In addition to proving the convergence of the error signal ε, the analysis also establishes a trigger condition associated with a trigger function that establishes when agents communicate state information.

To facilitate the subsequent convergence analysis, the event time t_k^i is explicitly defined below before proving the main theorem.

Definition 2. An event-triggered time t_k^i is defined as

$$t_k^i\triangleq\inf\left\{t>t_{k-1}^i\mid f_i\left(t\right)=0\right\},\,i\in\mathcal{V}, \tag{21}$$

for $k=1,2,\ldots$, where $t_0^i=0$, and $f_i\left(\cdot\right)$, denoted as $f_i\left(e_i\left(\cdot\right),\,\hat{z}_i\left(\cdot\right)\right)$, is a decentralized trigger function defined as

$$f_i\left(e_i,\,\hat{z}_i\right)\triangleq\left\|e_i\right\|-\sqrt{\frac{\eta_i\left(k_1-\frac{k_3}{\beta}\right)}{k_2+k_3\beta}}\left\|\hat{z}_i\right\|, \tag{22}$$

where $\eta_i\in\mathbb{R}_{>0}$ satisfying $0<\eta_i<1$ provides flexibility in real-time scheduling, and $\beta\in\mathbb{R}_{>0}$ is a positive constant satisfying

$$\beta>\frac{k_3}{k_1}, \tag{23}$$

where k_i, $i=1,\,2,\,3$, are positive constants defined as

$$\begin{aligned} k_1 &\triangleq \min_{p\in\mathcal{P}}\left\{\delta_{m1}S_{\min}\left(H_p^{-2}\right)\right\} \\ k_2 &\triangleq \max_{p\in\mathcal{P}}\left\{S_{\max}\left(H_p\otimes\left(2PBB^TP\right)\right)-\delta_{m1}\right\} \\ k_3 &\triangleq \max_{p\in\mathcal{P}}\left\{S_{\max}\left(I_N\otimes\left(2PBB^TP\right)-H_p^{-1}\otimes 2\delta_{m1}I_n\right)\right\}, \end{aligned}$$

where $\delta_{m1}\in\mathbb{R}_{>0}$ satisfies $0<\delta_{m1}<\delta_{\min}$ such that $k_2>0$ and $k_3>0$, and $S_{\min}\left(\cdot\right)$ and $S_{\max}\left(\cdot\right)$ denote the minimum and maximum singular value of the matrix argument, respectively.

Theorem 1. *The controller designed in (5) ensures that the network system in (1) and (2) modeled by the switching graph $\bar{\mathcal{G}}_\sigma$ achieves asymptotic leader-follower consensus defined in (4) provided that the estimate $\hat{x}_i$ in (5) is updated at t_k^i defined in Definition 2.*

Proof: Consider a Lyapunov function candidate $V:\mathbb{R}^{nN}\to\mathbb{R}$ defined as

$$V\triangleq\varepsilon^T\left(I_N\otimes P\right)\varepsilon, \tag{24}$$

where P is a symmetric positive definite matrix satisfying (12). Using (13) and (16), the time derivative of (24) can be expressed as

$$\begin{aligned} \dot{V} =& \varepsilon^T\left[I_N\otimes\left(PA+A^TP\right)-H_\sigma\otimes\left(2PBB^TP\right)\right]\varepsilon \\ &-e^T\left[H_\sigma\otimes\left(2PBB^TP\right)\right]\varepsilon. \end{aligned} \tag{25}$$

Since $H_{\sigma\in\mathcal{P}}$ is symmetric and positive definite, (12) and (14) can be used to upper bound (25) as

$$\dot{V}\leq-\delta_{\min}\varepsilon^T\varepsilon-e^T\left[H_\sigma\otimes\left(2PBB^TP\right)\right]\varepsilon. \tag{26}$$

Using (19), (26) can be expressed as

$$\begin{aligned} \dot{V}\leq& -\delta_{m1}\hat{z}^T\left(H_\sigma^{-2}\otimes I_n\right)\hat{z}-\delta_{m1}e^Te \\ &+2\delta_{m1}e^T\left(H_\sigma^{-1}\otimes I_n\right)\hat{z}-e^T\left[I_N\otimes\left(2PBB^TP\right)\right]\hat{z} \\ &+e^T\left[H_\sigma\otimes\left(2PBB^TP\right)\right]e-\delta_{m2}\varepsilon^T\varepsilon, \end{aligned} \tag{27}$$

where $\delta_{m2}\in\mathbb{R}_{>0}$ satisfies $\delta_{\min}=\delta_{m1}+\delta_{m2}$. By using the inequality $x^Ty\leq\left\|x\right\|\left\|y\right\|$, (27) can be upper bounded as

$$\dot{V}\leq-k_1\left\|\hat{z}\right\|^2+k_2\left\|e\right\|^2+2k_3\left\|e\right\|\left\|\hat{z}\right\|-\delta_{m2}\varepsilon^T\varepsilon. \tag{28}$$

Using the inequality $\left\|x\right\|\left\|y\right\|\leq\frac{\beta}{2}\left\|x\right\|^2+\frac{1}{2\beta}\left\|y\right\|^2$, (28) can be upper bounded as

$$\begin{aligned} \dot{V}\leq& -k_1\left\|\hat{z}\right\|^2+2k_3\left(\frac{\beta}{2}\left\|e\right\|^2+\frac{1}{2\beta}\left\|\hat{z}\right\|^2\right)+k_2\left\|e\right\|^2 \\ &-\delta_{m2}\varepsilon^T\varepsilon \\ \leq& -\sum_{i\in\mathcal{V}}\left[\left(k_1-\frac{k_3}{\beta}\right)\left\|\hat{z}_i\right\|^2-\left(k_2+k_3\beta\right)\left\|e_i\right\|^2\right] \\ &-\delta_{m2}\varepsilon^T\varepsilon. \end{aligned} \tag{29}$$

In (29), two necessary conditions for $\dot{V}$ to be negative definite are

$$0 < k_1 - \frac{k_3}{\beta}$$
$$\|e_i\|^2 \leq \frac{\eta_i \left(k_1 - \frac{k_3}{\beta}\right)}{k_2 + k_3\beta} \|\hat{z}_i\|^2, \tag{30}$$

which are satisfied provided that (21)-(23) are satisfied. Provided (30) and (23) are satisfied, then (29) can be rewritten as

$$\dot{V} \leq -\sum_{i\in\mathcal{V}} (1-\eta_i)\left(k_1 - \frac{k_3}{\beta}\right)\|\hat{z}_i\|^2 - \delta_{m2}\varepsilon^T\varepsilon, \tag{31}$$
$$\leq -\delta_{m2}\varepsilon^T\varepsilon, \tag{32}$$

which implies V is a common Lyapunov function. The linear differential inequality resulting from (24) and (32) can be solved to conclude that

$$\|\varepsilon\| \leq \|\varepsilon(0)\| \exp(-\gamma t),$$

where $\gamma \in \mathbb{R}_{>0}$ is a positive constant. The exponential convergence of $\|\varepsilon\|$ implies (4). ■

Remark 2. Based on (22), the constant c_i in Fig. 1 can be designed as $c_i = \sqrt{\frac{\eta_i\left(k_1 - \frac{k_3}{\beta}\right)}{k_2 + k_3\beta}}$. At t_k^i, e_i will be reset to zero as agent i communicates its state x_i to all its neighboring agents to update $\hat{x}_i$, and hence $\frac{\|e_i\|}{\|\hat{z}_i\|} = 0$ (i.e., $f_i < 0$). After the update, $\|e_i\|$ grows in time until meeting the next trigger condition $\frac{\|e_i\|}{\|\hat{z}_i\|} = c_i$ (i.e., $f_i = 0$). Then, the cycle repeats.

V. No Zeno Execution

Zeno execution is defined as infinite switches in a finite interval. Exclusion of Zeno execution can be sufficiently proven by finding a positive lower bound between any two contiguous discrete events (i.e., $\left[t_E^j, t_{E+1}^j\right)$) [10]. Based on Assumption 3, graph switches never cause Zeno execution (i.e., $\mu < t_{q+1} - t_q$). Therefore, only the following three intervals smaller than μ are analyzed.

Case 1. Consider any inter-event interval $\left[t_k^j, t_{k+1}^j\right)$, where $0 < t_{k+1}^j - t_k^j < \mu$. This interval is proven to be lower bounded by a positive constant in Theorem 2.

Case 2. Consider any inter-event interval $\left[t_q, t_{k+1}^j\right)$, for $0 < t_{k+1}^j - t_q < \mu$. By (9), a new neighbor agent $j \in \mathcal{N}_i$ has a mutual communication at t_q, at which time e_j is reset to zero. Therefore, t_q can be considered as the instant t_k^j, which implies *Case 1* and *Case 2* are equivalent.

Case 3. Consider any inter-event interval $\left[t_k^j, t_q\right)$, for $0 < t_q - t_k^j < \mu$. Then, the next cycle must be $\left[t_q, t_{k+1}^j\right)$ since t_{k+1}^j comes earlier than t_{q+1}. Therefore, proving a positive lower bound of the interval $\left[t_q, t_{k+1}^j\right)$ implies no Zeno execution since infinite switches can not happen in the finite interval. Moreover, finding the lower bound of $\left[t_q, t_{k+1}^j\right)$ is equivalent to proving *Case 2.*

Based on the three cases above, Zeno execution can be excluded provided that Theorem 2 is proven. To facilitate the subsequent analysis, two constants $\bar{c}_0, \bar{c}_1 \in \mathbb{R}_{>0}$ are defined as

$$\bar{c}_0 \triangleq \max_{p\in\mathcal{P}} \{S_{\max}(A)\} \tag{33}$$
$$\bar{c}_1 \triangleq \max_{p\in\mathcal{P}} \{S_{\max}((I_N \otimes A) + (H_p \otimes BK)) + S_{\max}(H_p \otimes BK) + S_{\max}(A)\}. \tag{34}$$

Theorem 2. *The event-triggered time defined in (21) ensures that there exists an agent $h \in \mathcal{V}$ such that the interval $\left[t_k^h, t_{k+1}^h\right)$ is lower bounded by*

$$\tau \geq \frac{1}{\max\{\bar{c}_0, \bar{c}_1\}} \ln\left(\frac{1}{N}\sqrt{\frac{\eta_h\left(k_1 - \frac{k_3}{\beta}\right)}{(k_2 + k_3\beta)}} + 1\right),$$

where $\tau \triangleq t_{k+1}^h - t_k^h \in \mathbb{R}_{>0}$ is the minimum interval, h is an agent that satisfies

$$h = \arg\max_{i\in\mathcal{V}} \sup_{t\in\mathbb{R}_{\geq 0}} \|\hat{z}_i\|.$$

Proof: Since $\|e_h\| \leq \|e\|$, the following inequality holds [10]

$$\frac{\|e_h\|}{N\|\hat{z}_h\|} \leq \frac{\|e\|}{N\|\hat{z}_h\|} \leq \frac{\|e\|}{\|\hat{z}\|},$$

which is equivalent to

$$\frac{\|e_h\|}{\|\hat{z}_h\|} \leq N\frac{\|e\|}{\|\hat{z}\|}. \tag{35}$$

For any interval $t \in \left[t_k^h, t_{k+1}^h\right)$, $\frac{\|e\|}{\|\hat{z}\|}$ is continuous. To show the inter-event interval is lower bounded as in [8], one can investigate the time derivative of $\frac{\|e\|}{\|\hat{z}\|}$ over the interval $t \in \left[t_k^h, t_{k+1}^h\right)$ as

$$\frac{d}{dt}\left(\frac{\|e\|}{\|\hat{z}\|}\right) = \frac{d}{dt}\left[\frac{\left(e^Te\right)^{\frac{1}{2}}}{\left(\hat{z}^T\hat{z}\right)^{\frac{1}{2}}}\right] \leq \frac{\|\dot{e}\|}{\|\hat{z}\|} + \frac{\|e\|\left\|\dot{\hat{z}}\right\|}{\|\hat{z}\|^2}. \tag{36}$$

Substituting $\dot{e}$ and $\dot{\hat{z}}$ from (11) and (20) into (36) yields

$$\frac{d}{dt}\left(\frac{\|e\|}{\|\hat{z}\|}\right) \leq \frac{\|(I_N \otimes A)e + (H_\sigma \otimes BK)\varepsilon + (H_\sigma \otimes BK)e\|}{\|\hat{z}\|} + \frac{\|e\|\|(I_N \otimes A)\hat{z}\|}{\|\hat{z}\|^2}.$$

Using (19) to replace ε and applying the inequality $x^Ty \leq \|x\|\|y\|$ yields

$$\frac{d}{dt}\left(\frac{\|e\|}{\|\hat{z}\|}\right) \leq \|(I_N \otimes A) + (H_\sigma \otimes BK)\|\frac{\|e\|}{\|\hat{z}\|} + \|(I_N \otimes BK)\| + \|(H_\sigma \otimes BK)\|\frac{\|e\|}{\|\hat{z}\|} + \|(I_N \otimes A)\|\frac{\|e\|}{\|\hat{z}\|},$$

which can be further expressed as

$$\dot{y} \leq \max\{\bar{c}_0, \bar{c}_1\}(1+y), \quad (37)$$

where $y : [0, \infty) \to \mathbb{R}_{\geq 0}$ is a non-negative, piecewise continuous function, which is differentiable at the inter-event interval and is defined as

$$y\left(t - t_k^h\right) \triangleq \frac{\|e(t)\|}{\|\hat{z}(t)\|}, \text{for}\, t \in \left[t_k^h, t_{k+1}^h\right) \quad (38)$$

for $k = 0, 1, 2 \ldots$. Based on (37), a non-negative function $\phi : [0, \infty) \to \mathbb{R}_{\geq 0}$, satisfying the differential equation

$$\dot{\phi} = \max\{\bar{c}_0, \bar{c}_1\}(1+\phi), \quad \phi(0) = y_0, \quad (39)$$

can be lower bounded by y as

$$y \leq \phi, \text{for}\, t \in [0, \tau), \quad (40)$$

where $y_0 \in \mathbb{R}_{\geq 0}$ is the initial condition of y, which is 0 since $e\left(t_k^h\right) = 0$. An analytical solution to (39) with initial condition $\phi(0) = 0$ can be solved as

$$\phi(t) = \exp(\max\{\bar{c}_0, \bar{c}_1\}t) - 1. \quad (41)$$

Using (40) and (41) with $t \to \tau$ yields

$$\lim_{\varphi \to 0^+} y(\tau - \varphi) \leq \exp(\max\{\bar{c}_0, \bar{c}_1\}\tau) - 1. \quad (42)$$

Using (22) where $f_h\left(t_{k+1}^h\right) = 0$, (35), and (38) yields

$$\frac{1}{N}\sqrt{\frac{\eta_h\left(k_1 - \frac{k_3}{\beta}\right)}{k_2 + k_3\beta}} \leq \exp(\max\{\bar{c}_0, \bar{c}_1\}\tau) - 1,$$

which implies that τ is lower bounded by a positive constant

$$\tau \geq \frac{1}{\max\{\bar{c}_0, \bar{c}_1\}} \ln\left(\frac{1}{N}\sqrt{\frac{\eta_h\left(k_1 - \frac{k_3}{\beta}\right)}{k_2 + k_3\beta}} + 1\right). \quad (43)$$

This lower bound implies that Zeno behaviors can be excluded. However, there is a trade-off between the minimum inter-event interval and the error convergence rate. The lower bound in (43) can be increased by selecting a higher η_h, but this increase results in a slower convergence due to the fact that $\dot{V}$ in (31) becomes less negative. Thus, agent h may adjust η_h to obtain either quicker convergence or less frequent communication. ■

VI. Discussion

A decentralized event-triggered control scheme for the leader-follower network consensus problem is developed to reduce communication with neighboring agents while ensuring the stability of the system. The estimate-based decentralized controller along with the decentralized trigger function reduces the number of inter-agent communications and prevents potential communication channel overload. A Lyapunov-based stability analysis indicates that the network system achieves asymptotic leader-follower consensus under this event-triggered control scheme. Moreover, the trigger function is proven to never exhibit Zeno behavior.

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